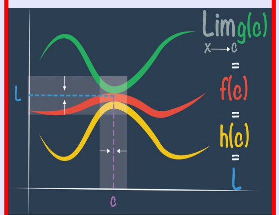
# Calculus I

# Lecture 39




Feb 19-8:47 AM

The Mean Value Theorem:

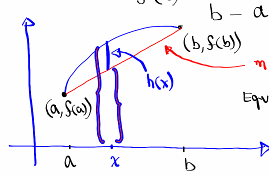
Let $f(x)$ be a function that meets the following conditions

1) $f(x)$ is continuous on $[a,b]$,

2) $f(x)$ is differentiable on $(a,b)$,

then there is a number $c$ in $(a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$




$m = \frac{f(b)-f(a)}{b-a}$

Equation of secant line

$y - f(a) = \frac{f(b)-f(a)}{b-a}(x-a)$

$$\boxed{y = \frac{f(b)-f(a)}{b-a}(x-a) + f(a)}$$

$h(x) = y_{curve} - y_{line}$

$h(x) = f(x) - \left[\frac{f(b)-f(a)}{b-a}(x-a) + f(a)\right]$

$h(x)$ is cont. on $[a,b]$.

$h(x)$ is diff. on $(a,b)$.

$h(a) = f(a) - \left[\frac{f(b)-f(a)}{b-a}(a-a) + f(a)\right] = f(a) - f(a) = 0$

$h(b) = f(b) - \left[\frac{f(b)-\cancel{f(a)}}{\cancel{b-a}}(\cancel{b-a}) + \cancel{f(a)}\right] = f(b) - f(b) = 0$

$h(a) = h(b)$

So by Rolle's thrm, there is a number $c$ in $(a,b)$ such that $h'(c) = 0$

Apr 23-8:46 AM

$$h(x) = f(x) - \left[\frac{f(b)-f(a)}{b-a}(x-a) + f(a)\right]$$

$$h(x) = f(x) - \frac{f(b)-f(a)}{b-a}(x-a) - f(a)$$

$$h'(x) = f'(x) - \frac{f(b)-f(a)}{b-a} \cdot 1 - 0$$

$$h'(x) = f'(x) - \frac{f(b)-f(a)}{b-a}$$

by Rolle's Thrm, $h'(c) = 0$

$$f'(c) - \frac{f(b)-f(a)}{b-a} = 0$$

Conclusion of MVT

$$\boxed{f'(c) = \frac{f(b)-f(a)}{b-a}}$$

Apr 23-9:02 AM

Verify the Conditions of MVT for $f(x) = 2x^2 - 3x + 1$ on $[0,2]$, then find $c$ that satisfies the Conclusion of MVT.

$f(x) = 2x^2 - 3x + 1$ is a polynomial function therefore it is cont. & diff. everywhere.

$f(0) = 1$

$f(2) = 2(2)^2 - 3(2) + 1 = 3$

$f'(x) = 4x - 3$

$f'(c) = 4c - 3$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$4c - 3 = \frac{3-1}{2-0} \qquad 4c - 3 = \frac{2}{2} \qquad 4c - 3 = 1$$

$\boxed{c = 1}$

is in $(0,2)$

Apr 23-9:06 AM

$\boxed{f'(x) \geq 2}$ on $(1,4)$, $\boxed{f(1) = 10}$, Discuss all possible values of $f(4)$.

Since $f'(x) \geq 2$ on $(1,4)$

$\Rightarrow f(x)$ is diff. & cont. on $(1,4)$

by MVT $\quad f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$$f'(c) = \frac{f(4) - f(1)}{4 - 1}$$

$$= \frac{f(4) - 10}{3} \geq 2$$

$$f(4) - 10 \geq 6$$

$$\boxed{f(4) \geq 16}$$

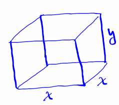
$f(4)$ is at least 16.

Apr 23-9:12 AM

A box has Square base, open top and Volume of 32000 $cm^3$.

$V = x \cdot x \cdot y = 32000$

$x^2 y = 32000$ $\quad y = \dfrac{32000}{x^2}$




$x > 0$, $y > 0$

Find dimensions of such box with minimum materials.

Material = Base + 4 Sides

$= x \cdot x + 4\, x \cdot y$

$m(x) = x^2 + 4x \cdot \dfrac{32000}{x^2}$

$m(x) = x^2 + \dfrac{128000}{x}$

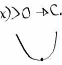
$m(x) = x^2 + 128000\, x^{-1}$

$m'(x) = 2x - 128000\, x^{-2}$ $\quad \to m'(x) = 2x - \dfrac{128000}{x^2}$

$m''(x) = 2 + 256000\, x^{-3}$ $\quad \to m''(x) = 2 + \dfrac{256000}{x^3}$

Since $x > 0$

$m''(x) > 0 \to$ C.U.



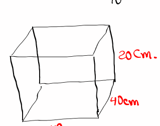
Min. where $m'(x) = 0$

Solving $m'(x) = 0$ $\quad 2x - \dfrac{128000}{x^2} = 0$

Multiply by $x^2$ and divide by 2

$x^3 - 64000 = 0 \to x = \sqrt[3]{64000} \to \boxed{x = 40}$

$y = \dfrac{32000}{40^2} = \dfrac{\cancel{32}^{2}0\cancel{00}}{\cancel{16}\cancel{00}} \to \boxed{y = 20}$




$V = 32000\ cm^3$

Materials with open top is minimum.

40cm by 40cm by 20cm

How much materials

Materials = base + 4 Sides

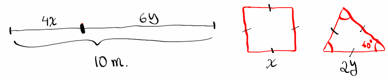
$= 40 \cdot 40 + 4 \cdot 40 \cdot 20$

$= 1600 + 3200$

$= 4800\ cm^2$

Apr 23-9:18 AM

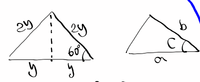
A piece of wire is 10m long. It is cut into two pieces. One piece is square and the other piece is an equilateral triangle.
How should we cut it to have total area enclose to be minimum and/or maximum?




$4x + 6y = 10$

Total Area = $A_{square} + A_{Triangle} =$

$x^2 + y^2\sqrt{3}$




$\to 6y = 10 - 4x$

$y = \frac{10-4x}{6} = \frac{5-2x}{3}$

Area $= \frac{1}{2} ab \operatorname{Sin} C$

$= \frac{1}{\cancel{2}} \cdot \cancel{2}y \cdot 2y \cdot \operatorname{Sin} 60°$

$= \cancel{2}\, y^2 \cdot \frac{\sqrt{3}}{\cancel{2}} = y^2\sqrt{3}$

$f(x) = x^2 + \left(\frac{5-2x}{3}\right)^2 \cdot \sqrt{3}$

$\frac{(5-2x)^2 \cdot \sqrt{3}}{9}$

$f'(x) = 2x + \frac{\sqrt{3}}{9}\left(2 \cdot (5-2x)^1 \cdot (-2)\right)$

$f'(x) = 2x - \frac{4\sqrt{3}}{9}(5-2x)$

$f''(x) = 2 - \frac{4\sqrt{3}}{9}(-2) = 2 + \frac{8\sqrt{3}}{9} > 0 \to f(x)$ is C.U.

$f'(x) = 0$

$2x - \frac{4\sqrt{3}}{9}(5-2x) = 0$

$18x - 4\sqrt{3}(5-2x) = 0$

$18x - 20\sqrt{3} + 8x = 0$

Also find $f(0)$ & $f(10)$

$26x = 20\sqrt{3}$

$x = \frac{20\sqrt{3}}{26}$ $\quad x = \frac{10\sqrt{3}}{13}$

Mistake on $f'(x)$

Apr 23-9:32 AM